For its probable future influence, its suggestive analyses, its rich lore of illuminating and interesting mathematical and archeological references, and not least for its exquisite half-tones of rarely charming vases, this book is recommended for the attention and study but not the blind acceptance of such as appreciate science and art.

ALBERT A. BENNETT.

Calculus and Graphs—Simplified for a First Brief Course. By L. M. PASSANO. New York, The Macmillan Company, 1921. 12mo. 8+167 pp. Price $1.75.

As every one is aware, who has had the pleasure of teaching the subject of calculus, many features of the subject present no essential difficulties even to students who have shown no special aptitude along mathematical lines. The mere technique of differentiation can be readily acquired by students who cannot learn facility in the manipulation of trigonometric identities. The chief difficulty in solving elementary problems seems to lie in the lack of ability displayed by the student in converting a verbally formulated problem into tractable equations. This is a difficulty that is earlier presented by the subject of algebra, and has nothing to do with any one particular branch of mathematics. As long as verbal problems are offered for analysis, there will be students who find the subject difficult. In the present book there has been no effort to spare the student in this direction. Since the development of clear reasoning is one of the primary purposes in any mathematical course, this book assuredly fulfils its principal aim.

Of the other difficulties which the subject sometimes presents may be mentioned the extensiveness of the field of applications, and questions of rigor connected with the notion of limit. In the text under discussion, the material is wisely restricted so as to make possible an emphasis upon essentials without which the student can receive no benefit from the course. The author gives us the following warning in his preface: "The aim has been to make the student understand the subject; not to write a book that would satisfy meticulous mathematical pedantry. In so doing the author may have in places sacrificed logical detail to simplicity of presentation, but never, he hopes, accuracy of statement. In the opinion of the writer a too rigidly logical proof with its paraphernalia of subscripted Greek letters is out of place in an elementary first course in calculus, for the reason that the student never understands such a proof. Or if by arduous effort he does grasp its meaning, it is at the expense—in time and labor—of other things that are more important and far more useful." In view of this announcement the reader need not be surprised to see infinite series used where necessary without a word about convergence; but some other features are not so readily explained. On page 15, we read, "We do not know whether $x + 2$ is plus or minus zero; that is, whether $x + 2$ is an exceedingly small positive or an exceedingly small negative number." On page 19, and repeatedly throughout the book, "The increment of the function can be made smaller than any number we may assign, however small, by making the increment of the independent variable small enough . . .," where the word "number" is apparently expected to mean

"positive number." On page 80, the phrase "any number of parts" is to be understood as meaning "any number of equal parts." The book is almost free of definitions. A search through the text has failed to reveal any definition or even hint as to what may be meant by "element" until on page 93, we read, "To make use of this new tool [the definite integral] all that is necessary is to form the element of the thing to be summed; to get a differential expression representing, in terms of some independent variable, any one of the terms to be summed. This element being found and formed, the remainder of the process consists in integrating, and substituting the limits as shown in (40)," while on page 122, we read "of another little element." The problems considered on maxima and minima are of the usual sort, and, as is obvious, examples may readily be constructed in which an extremal is obtained at the end of the interval of definition. That no such examples should happen to be incorporated in the particular collection here given does not justify the statement: "For the function to reach a maximum or a minimum value, must $dy/dx \doteq 0$. This, as we say, is a *necessary* condition for a maximum or minimum value of the function." One need hardly state that the notion of an interval of definition is not considered. The student is asked to *prove* that a locus is a circle, by plotting its points (p. 45, Ex. 26).

A more practical criticism, from the viewpoint of the text, is one of little logical import and refers to the form of the discussion. Answers obtained in the illustrative examples are not so labeled. The discussion consists in most cases of a mere sequence of equations with no hint as to what is assumed and what derived and whence. Many of the discussions continue uninterruptedly for too many pages for any normally impatient student to desire to understand them. This is particularly pertinent to section 20. What student will care to follow the advice: "Formula (63) can best be learned in words. Thus: The integral of the product of two factors, one of which can be integrated separately, *equals* the product of the integral of that one factor times the other; *minus* a new integral consisting of the same integral of the one factor times the derivative of the other"? The use made of units in a book intended for engineers, or rather the abuse of them, is a surprise—for example, "200 π radians minute," without even the apology of an abbreviation, stands for a velocity in radians per minute, and similarly throughout the book. The misprints are few. Only one bad case was noted, where a letter of the wrong font was used on page 124, 5th line. Something like half of the formulas appear with no terminal punctuation, so that this unusual procedure appears to be intentional, although not observed consistently.

Despite all of these incidental objections and many more that are trivial, the general arrangement of the material and the choice of exercises are such as to suggest that the book might prove a very satisfactory text in the increasing number of institutions that are giving first year students a taste of one of the richest treats afforded by mathematics, the calculus.

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